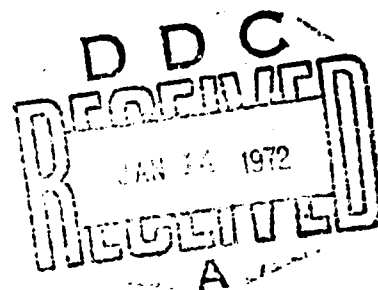


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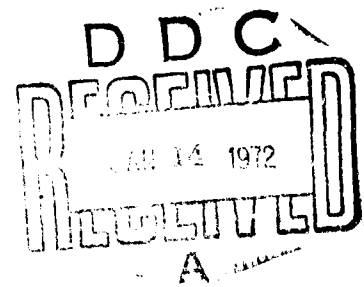
DIFFRACTION OF SURFACE WAVES
BY A SUBMERGED BODY IN MOTION

By

Peter Van Dyke and
Willem P. A. Joosen

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NOTATION

a	Alteration factor for incident wave height
a_m	Mean wave height alteration parameter
\bar{A}	Waterplane area of body
\tilde{A}	Longitudinal transverse area of body
A_n, B_n	Constants
b	Constant in wave spectrum representation
B	Beam of body
c	Propagating velocity of incident gravity waves
C	Contour of body transverse cross section
d	Ratio of body depth to body length
e	Base of natural logarithms
E, E_∞	Local and reference energy densities
f	Function defining transverse cross section of body
F	Unknown source strength at surface of body
Fr	Froude number with body depth as reference
g	Gravitational acceleration
g_n	Wave contribution in G_n
G	Green's function

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G_1, G_2	Portions of G
h	Dimensionless wave height parameter
H	Wave height
k	Dimensionless incident wave number
K	Function of ξ and $\bar{\xi}$
L	Length of body
n, m	Subscripts
p, p_{∞}	Local and reference wave amplitudes
q	Constant of integration
q_n	Poles
$S, S_{\bar{\xi}}, T$	Functions of horizontal and vertical longitudinal cross sections
t	Time
u, v	Stationary phase function constants
V	Uniform velocity of body
\hat{U}, \hat{V}	Stationary phase functions
\bar{U}, \bar{V}	Numerically evaluated functions for $\epsilon = 1 - 4\gamma$
\tilde{U}, \tilde{V}	Functions for $\epsilon = 4\gamma - 1$
w	Function of θ
x	Ratio of distance in axial direction from origin (midship) to body length

\bar{X}, \tilde{X}	Correction factors for U and V
y	Constant of integration
ξ, η, ζ	Coordinate system fixed to body
α	Angle of incident wave direction with negative ξ axis
β	Dimensionless horizontal cross section ordinate
γ	Dimensionless wave encounter parameter
δ	Dimensionless body velocity parameter
ϵ	Small parameter
θ	Constant of integration around body
λ	Wave length
σ	Frequency of wave encounter
τ	Dimensionless ordinate of lateral vertical cross section
ϕ	Disturbance of ϕ_w due to presence of body
ϕ_w	Dimensionless potential of incident wave
ϕ_0	Steady portion of dimensionless potential due to forward motion of body
ϕ_1	Wave portion of dimensionless potential ϕ
ϕ_T	Total dimensionless potential
ψ	Velocity potential
ω	Wave frequency in fixed co-ordinates

ABSTRACT

Surface waves at the water surface are influenced locally by a submerged moving slender body. In addition to the Kelvin wave pattern which would be created by the moving body when travelling at uniform speed under smooth water, waves on the surface can be diffracted; this report is concerned with the diffraction effect. It is assumed that the body is moving at constant speed, is not oscillating, and that the wave heights are small in comparison with the body dimensions, so that a linearized theory can be applied. Using slender body theory, relative wave heights can be evaluated. For monochromatic waves, two critical, singular points, at which the linearized theory does not hold, occur when the component of body velocity in the incident wave direction, and relative to the wave, is equal to the group velocity of the wave with negative or positive sign: at the former point, disturbances caused by the body, and existing ahead of the body, begin to grow to infinite amplitude; at the latter point, monochromatic waves behind the body propagate at the energy velocity of the wave, and are thus also of infinite amplitude. For the coincident wave-body direction case, approximate expressions for the results are obtained for three particular ranges of the encounter frequency and body velocity parameters. However, by considering a wave spectrum with a local spectral density, instead of a monochromatic wave, the singularities of the linear theory can be removed and a mean

increase in wave height can be defined over the entire encounter-velocity domain. The required integrals over the wave spectrum case are discussed; the actual numerical evaluation of these integrals has yet to be carried out.

FORMULATION OF THE PROBLEM

When a body moves beneath surface gravity waves, the question arises: to what extent is the propagating wave pattern affected by the presence of the body? In addition to the stationary wave pattern created by the moving body when travelling under smooth water, incident waves are disturbed by diffraction; this phenomenon is the subject of the present study. It is assumed that the body is slender and in uniform forward motion; the initial discussion deals with monochromatic surface waves travelling in a fixed direction.

The coordinate system is given in Figure 1; $L/2$ is the unit of length, L being the length of the body. The coordinate system (ξ, η, ζ) is fixed in the body and moves at speed V in the positive ξ direction. The contour $C(\zeta)$ of the transverse vertical cross section is described by the function $\eta = f(\xi, \zeta)$. The wave heights are assumed to be small compared to the body dimensions even when the body is near the free surface; consequently, a linearized theory can be applied.

Since the fluid can be assumed to be incompressible, inviscid and irrotational, the problem can be described by a velocity potential ϕ , or, in a dimensionless notation

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$$\varphi_T = \sigma \frac{1}{g} L \quad [1]$$

where

$$\varphi_T = \gamma \varphi_0 + \varphi_w e^{-i\sigma t} + \varphi e^{-i\sigma t} \quad [2]$$

with σ the frequency of wave encounter, φ_0 the steady part of the potential due to the forward motion of the body, φ_w the potential of the incident wave, and φ characterizing the disturbance of φ_w due to the presence of the body. φ_0 , φ_w , and φ are all time independent, and the encounter parameter γ is given by

$$\gamma = \sigma V/g \quad [3]$$

The nondimensional wave potential φ_w is given by

$$\varphi_w(\xi, \eta, \zeta) = h e^{k(\zeta - 2d) + ik(\xi \cos \alpha + \eta \sin \alpha)} \quad [4]$$

with α the angle of the incident wave direction with the negative ξ axis, d the body depth divided by L , and h the wave height parameter given by

$$h = \sigma H/\omega L \quad [5]$$

with H the wave height, and ω the wave frequency in a coordinate system fixed in space. The non-dimensional wave number k is given by

$$k = \omega^2 L / 2g = \pi L / \lambda \quad [6]$$

with λ the wave length. The relationship between σ and ω is given by

$$\sigma = \omega - V\omega^2 \cos \alpha / g \quad [7]$$

By defining

$$\delta = \omega V / g = Fr(2kd)^{1/2} = V/c \quad [8]$$

with the Froude number based on depth

$$Fr = V / (gLd)^{1/2} \quad [9]$$

the wave encounter parameter γ can be related to the velocity parameter δ by

$$\gamma = \delta - \delta^2 \cos \alpha \quad [10]$$

The wave encounter problem is discussed in Reference 1 with particular regard for the interpretations to be given to encounter and velocity parameters so that a wave potential of the form given by Equation [4] can be assumed to hold throughout the $\sigma - \alpha$ plane. These results can be summarized as: the velocity V is always ≥ 0 ; for $\delta \cos \alpha < 1$, the encounter frequency is equal to σ by Equation [7], and the encounter angle is α ; for $\delta \cos \alpha > 1$, the encounter frequency is equal to $-\sigma$,

the encounter angle is $\alpha + \pi$, and the wave encounter parameter given by Equation [10] changes sign so as to remain ≥ 0 . These interpretations will be discussed later when the particular case of coincident encounter ($\alpha = 0, \pi$) is considered.

There are several solutions to the problem of wave diffraction over a submerged cylinder, with the waves travelling normally (Reference 2) and obliquely (Reference 3) to an infinite cylinder. The solution given in the present report employs slender body theory and assumes that the (finite) body has a fine shape at the bow and stern; it is thus particularly well suited to the case of co-incident wave-body direction, but not as applicable to the normal encounter case.

A theoretical treatment of the slender body diffraction problem is given in Reference 4, but assumptions restricting the encounter angle to 0 and π , and the body velocity to very small values, limit its applicability.

A detailed statistical evaluation of surface wave diffraction is outlined in Reference 5, but the required scattering function to which the method would be applied is not discussed in any detail. The approach taken in the present report is aimed at, first, obtaining the scattering behavior, and then treating the statistics of waves in a simplified manner.

THE GENERAL SOLUTION

Following the approach of the hydrodynamic theory of ship motion, (References 6, 7) the function φ may be written as

$$\varphi(\bar{\xi}, \bar{\eta}, \bar{\zeta}) = \int_{-1}^1 d\xi \int_C d\zeta F(\xi, \zeta) G(1 + (f, \xi)^2 + (f, \zeta)^2)^{\frac{1}{2}} \quad [11]$$

where $(\bar{\xi}, \bar{\eta}, \bar{\zeta})$ is the point in the (ξ, η, ζ) co-ordinate system where the wave potential is to be determined, $F(\xi, \zeta)$ is the unknown source strength at the body surface, and $G(\bar{\xi}, \bar{\eta}, \bar{\zeta}; \xi, \eta, \zeta)$ is the Green's function which satisfied the Laplace equation, the free surface condition, and the radiation condition. The function F is determined from the boundary condition at the body

$$\frac{\partial \varphi}{\partial \eta} = - \frac{\partial \varphi_w}{\partial \eta} \quad [12]$$

The function G is discussed in References 6 and 7.

Approximate expressions for φ will be derived in the following paragraphs for the case of a slender body.

The Slender Body Approximation

In slender body theory, the transverse dimensions of the body are assumed to be small with respect to the length of the body. Using this assumption, G behaves at the body surface as

$$G \approx 1/((\bar{\xi}-\xi)^2 + (\bar{\eta}-\eta)^2 + (\bar{\zeta}-\zeta)^2)^{\frac{1}{2}} + K(\bar{\xi}, \xi) \quad [13]$$

where K is a function only of $\bar{\xi}$ and ξ . Using Equations [4] and [11], the boundary condition of Equation [12] becomes

$$\begin{aligned} -4\pi F(\bar{\xi}, \bar{\zeta}) - 2 \frac{\partial}{\partial \eta} \oint_C d\zeta F(\xi, \zeta) \ln ((\bar{\eta}-f)^2 + (\bar{\zeta}-\zeta)^2)^{\frac{1}{2}} (1 + (f, \bar{\zeta})^2)^{\frac{1}{2}} = \\ = -kh(1 \sin \alpha \operatorname{sgn}(\bar{\xi}) + (f, \bar{\xi}) \operatorname{sgn}(\bar{\eta})) e^{-2kd + 1k\bar{\xi} \cos \alpha} / (1 + (f, \bar{\xi})^2)^{\frac{1}{2}} \end{aligned} \quad [14]$$

Integration along the cross section contour with respect to $\bar{\xi}$ gives the functions

$$\begin{aligned} S(\bar{\xi}) &= \int_C d\bar{\zeta} F(\bar{\xi}, \bar{\zeta}) (1 + (f, \bar{\zeta})^2)^{\frac{1}{2}} \\ &= hk \left[\beta(\bar{\xi}) + i\tau(\bar{\xi}) \sin \alpha \right] e^{-2kd + 1k\bar{\xi} \cos \alpha} / 2\pi \end{aligned} \quad [15]$$

$$S, \bar{\xi}(\bar{\xi}) = 1kS(\bar{\xi}) \cos \alpha + T(\bar{\xi}) \quad [16]$$

$$T(\bar{\xi}) = hk \left[\beta, \bar{\xi} + i\tau, \bar{\xi} \sin \alpha \right] e^{-2kd + 1k\bar{\xi} \cos \alpha} / 2\pi \quad [17]$$

where $\beta(\bar{\xi})$ is the dimensionless horizontal cross-section offset, and $\tau(\bar{\xi})$ the dimensionless lateral vertical cross-section offset.

The wave height is determined from the behavior of the potential ϕ at the free surface. At $\bar{\xi} = 2d$ the function G (see Reference 7) behaves as

$$G \approx 2/((\bar{\xi}-\xi)^2 + \bar{\eta}^2 + 4d^2)^{\frac{1}{2}} + G_1 + G_2, \xi \quad [18]$$

with

$$\begin{aligned}
 -\pi G_n/2 = & \int_0^{\theta_1} d\theta \int_0^\infty dq \frac{A_n(q, \theta) e^{-2qd+1q(\bar{\xi}-\xi) \cos \theta} \cos(\bar{\eta}q \sin \theta)}{(\delta^2 q^2 \cos^2 \theta/k) + 2\gamma q \cos \theta + (\gamma^2 k/\delta^2) - q} + \\
 & + \int_{\theta_1}^{\pi/2} d\theta \int_{M_1}^\infty dq \frac{A_n(q, \theta) e^{-2qd+1q(\bar{\xi}-\xi) \cos \theta} \cos(\bar{\eta}q \sin \theta)}{(\delta^2 q^2 \cos^2 \theta/k) + 2\gamma q \cos \theta + (\gamma^2 k/\delta^2) - q} + \\
 & + \int_0^{\pi/2} d\theta \int_{M_2}^\infty dq \frac{B_n(q, \theta) e^{-2qd+1q(\bar{\xi}-\xi) \cos \theta} \cos(\bar{\eta}q \sin \theta)}{(\delta^2 q^2 \cos^2 \theta/k) - 2\gamma q \cos \theta + (\gamma^2 k/\delta^2) - q} \quad [19]
 \end{aligned}$$

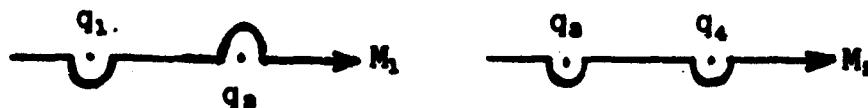
where

$$\begin{aligned}
 A_1 &= B_1 = \gamma^2 k/\delta^2 \\
 A_2 &= -1((\delta^2 q \cos \theta/k) + 2\gamma) \\
 B_2 &= 1((\delta^2 q \cos \theta/k) - 2\gamma) \quad [20]
 \end{aligned}$$

and

$$\theta_1 = \cos^{-1} (1/4\gamma) \quad [21]$$

The integration contours M_1 and M_2 in the q plane are given by



The poles q_1 , q_2 , q_3 , and q_4 of the integrands are

$$\begin{aligned} q_{1,2} &= k(1-2\gamma \cos \theta \mp (1-4\gamma \cos \theta)^{\frac{1}{2}})/2b^2 \cos^2 \theta \\ q_{3,4} &= k(1+2\gamma \cos \theta \mp (1+4\gamma \cos \theta)^{\frac{1}{2}})/2b^2 \cos^2 \theta \end{aligned} \quad [22]$$

Assuming $S(1) = S(-1) = 0$, the potential ϕ at the free surface can be written as

$$\begin{aligned} \phi(\bar{\xi}, \bar{\eta}, 2d) &= \int_{-1}^1 d\xi \, 2S(\xi)/((\bar{\xi}-\xi)^2 + \bar{\eta}^2 + 4d^2)^{\frac{1}{2}} + \\ &+ \int_{-1}^1 d\xi \, S(\xi) \, G_1 + \int_{-1}^1 d\xi \, S(\xi) \, G_2 \end{aligned} \quad [23]$$

The Wave System

Since only the wave pattern is of interest here, the wave portion, $\tilde{\phi}$, of the potential ϕ , will be considered. The wave contribution of G_n , denoted by g_n , emanates from the residue of the four poles. Using Equations [15], [16], [17] and [23], and integrating by parts, $\tilde{\phi}$ becomes

$$\tilde{\varphi}(\bar{\xi}, \bar{\eta}, 2d) = \int_{-1}^1 d\xi S(\xi) (g_1 + ik g_2 \cos \alpha) + \int_{-1}^1 d\xi T(\xi) g_2 \quad [24]$$

where g_n is defined, for $\xi - \bar{\xi} < 0$, in front of the body,

$$g_n = 4i \int_{\theta_1}^{\pi/2} d\theta A_n(q_1(\theta), \theta) e^{-q_1(\theta)(2d-1(\bar{\xi}-\xi) \cos \theta)} \times \\ \times \cos(\bar{\eta} q_1(\theta) \sin \theta) / (1-4\gamma \cos \theta)^{\frac{1}{2}} \quad [25]$$

and for $\xi - \bar{\xi} > 0$, behind the body,

$$g_n = 4i \int_{\theta_1}^{\pi/2} d\theta A_n(q_2(\theta), \theta) e^{-q_2(\theta)(2d+1(\bar{\xi}-\xi) \cos \theta)} \cos(\bar{\eta} q_2(\theta) \sin \theta) / (1-4\gamma \cos \theta)^{\frac{1}{2}} + \\ + 4i \int_0^{\pi/2} d\theta B_n(q_3(\theta), \theta) e^{-q_3(\theta)(2d-1(\bar{\xi}-\xi) \cos \theta)} \cos(\bar{\eta} q_3(\theta) \sin \theta) / (1+4\gamma \cos \theta)^{\frac{1}{2}} - \\ - 4i \int_0^{\pi/2} d\theta B_n(q_4(\theta), \theta) e^{-q_4(\theta)(2d-1(\bar{\xi}-\xi) \cos \theta)} \cos(\bar{\eta} q_4(\theta) \sin \theta) / (1+4\gamma \cos \theta)^{\frac{1}{2}} \quad [26]$$

From this it can be concluded that one wave is progressing in front of the body and the other three are behind the body.

The existence of this wave pattern can easily be illustrated by considering the two-dimensional problem of a line source at the surface of the water (see References 7 and 8). It can also be understood by a rather simple consideration: In a coordinate system fixed in space, gravity waves must satisfy the dispersion relationship, Equation [6],

$$\omega^2 = 2gk/L \text{ or } c^2 = gL/2k \quad [27]$$

In a coordinate system moving at speed V in the direction (plus or minus) of the waves the frequency is given by

$$\sigma = 2k(c \pm V)/L = 2gk/L \pm 2kV/L \quad [28]$$

This relation shows that four waves exist, which have the same frequency σ , but different wavelengths.

Range of Parameters

As an illustration of the solution regimes, and the domains of the encounter and frequency parameters, the case of body and wave in the same direction will be considered. This implies an α value of 0 or π .

There are then three domains of interest covering the cases of the body heading into the waves, the waves overtaking the body, and the body overtaking the waves. In the first domain, the angle α is π , the body is heading into the waves, $\delta \cos \alpha$ is always less than 0, and

$$\gamma = \delta + \delta^2, \alpha = \pi \quad [29]$$

The second domain occurs where, with $\alpha = 0$, $0 \leq \delta \leq 1$, and

$$\gamma = \delta - \delta^2, \alpha = 0 \quad [30]$$

In the third domain, with $\alpha = 0$ and $\delta \geq 1$, the body is overtaking the waves, and redefinition of encounter parameters implies

$$\gamma = -\delta + \delta^2, \alpha = \pi \quad [31]$$

In the first domain, γ increases steadily from 0 as δ increases from 0; the point $\gamma = 1/4$, which will become significant in subsequent paragraphs, occurs at δ of $((2)^{1/2} - 1)/2$. In the second domain, γ increases from 0 to a peak value of $1/4$ at $\delta = 1/2$; then decreases back to zero; it is symmetric about $\delta = 1/2$. The third regime has γ increasing with $(\delta-1)$ in the same way as in region one.

APPROXIMATE SOLUTIONS

In order to obtain a qualitative solution for the wave form, approximate evaluations of the integrals of Equations [25] and [26] can be carried out where the point of interest is far from the body ($|\xi| \gg 1$).

Stationary Phase

The method of stationary phase may be applied to the integrals in Equations [25] and [26], as long as all functions except the exponential behave smoothly near the stationary point. Using the series representations of Reference 9, the stationary point at $\theta = 0$ yields the results, for $\gamma > 1/4$, $\xi - \bar{\xi} < 0$,

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$$g_n = 0 \quad [32]$$

and for $\xi - \bar{\xi} > 0$

$$g_n = \hat{U}(q_3, 0, B_n, -1, +3\pi/4) + \hat{U}(q_4, 0, B_n, -1, -\pi/4) \quad [33]$$

where

$$\hat{U}(q_m(\theta), \theta, B_n, u, v) = \frac{u^{1/4}(\pi)^{1/2} B_n(q_m(\theta), \theta) e^{-q_m(\theta)(2d+iu(\xi-\bar{\xi}))+iv}}{(2q_m(\theta)(1-4u\gamma)^{1/2} |\bar{\xi}|)^{1/2}} \quad [34]$$

The poles q_m are given by Equation [22]. For $\gamma > 1/4$, q_1 and q_2 have imaginary parts and thus produce an exponential decay which results in their omission.

The next term in the g_n solution series at the $\theta = 0$ stationary point is of order $|\bar{\xi}|^{-3/2}$. At the $\theta = \pi/2$ point, however, a term of order $|\bar{\xi}|^{-1}$ exists; Equation [33] then has an additional term

$$14B_n(q_3(\pi/2), \pi/2) e^{-2q_3(\pi/2)d} \cos(\bar{\eta}q_3(\pi/2))/q_3(\pi/2) |\bar{\xi}| \quad [35]$$

before terms of $|\bar{\xi}|^{-3/2}$ appear. This result is of interest because it shows that the wave diffraction phenomenon is predominant along the axis of the body ($\bar{\eta} = 0$), with off-axis disturbances decaying as $|\bar{\xi}|^{-1}$. In all subsequent results, however, only the leading terms of order $|\bar{\xi}|^{-1/2}$ will be carried.

The wave diffraction potential $\tilde{\phi}$ can now be written, for $\gamma > 1/4$ and $\bar{\xi} > 0$, as

$$\tilde{\phi} = 0 \quad [36]$$

while for $\bar{\xi} < 0$,

$$\tilde{\phi} = \hat{V}(q_4(0), -1, -3\pi/4) + \hat{V}(q_3(0), -1, +\pi/4) \quad [37]$$

with

$$\hat{V}(q_m(0), u, v) = \frac{khe^{-2d(k+q_m(0)) + 1u\bar{\xi}q_m(0) + 1v}}{(\pi|\bar{\xi}|/2q_m(0))^{\frac{1}{2}}(1-4u\gamma)^{\frac{1}{4}}} \times$$

$$\times \int_{-1}^1 d\xi (\beta + i\tau \sin \alpha) e^{1(k \cos \alpha - uq_m(0))\xi} \quad [38]$$

For the case where $\gamma < 1/4$, the poles q_1 and q_2 contribute, leading to, for $\xi - \bar{\xi} < 0$,

$$g_n = \hat{U}(q_1, 0, A_n, +1, -\pi/4) \quad [39]$$

and for $\xi - \bar{\xi} > 0$,

$$g_n = \hat{U}(q_2, 0, A_n, +1, +\pi/4) + \hat{U}(q_3, 0, B_n, -1, +3\pi/4) +$$

$$+ \hat{U}(q_4, 0, B_n, -1, -\pi/4) \quad [40]$$

The wave diffraction potential now becomes, for $\gamma < 1/4$ and $\bar{\xi} > 0$,

$$\tilde{\phi} = \hat{V}(q_1(0), +1, +\pi/4) \quad [41]$$

and for $\bar{\xi} < 0$,

$$\tilde{\phi} = \hat{V}(q_2(0), +1, +3\pi/4) + \hat{V}(q_4(0), -1, -3\pi/4) + \hat{V}(q_3(0), -1, +\pi/4) \quad [42]$$

The cross-sectional area integral in Equation [38] for \hat{V} can be clarified by the relationships

$$\int_{-1}^1 d\xi \beta(\xi) = 4\bar{A}/L^2 \quad \int_{-1}^1 d\xi \tau(\xi) = 4\tilde{A}/L^2 \quad [43]$$

where \bar{A} is the longitudinal-horizontal cross-sectional area of the body, and \tilde{A} the longitudinal-vertical cross-sectional area.

Several qualitative results can be distilled from the general results obtained so far. For example, in the specific case where d is large enough so that $e^{-2q_3d} \gg e^{-2q_2d}$, the body is moving directly into the waves, with $\alpha = \pi$, and $1-4\gamma$ is not small, the term $\hat{V}(q_3, -1, +\pi/4)$ dominates the wave potential for $\bar{\xi} < 0$ and values of $\gamma > 1/4$. By comparing the wave potential $\tilde{\phi}$, at a specific point $\bar{\xi} = -2x$ behind the body, the result

$$\phi_1(-2x, 2d) = ae^{+\pi i/4} \phi_w(-2x, 2d) \quad [44]$$

is obtained, in which a can be regarded as an incident wave height alteration factor, given by

$$a = 4k^{3/2} e^{-4dk} \bar{A}/L^2 (\pi^2 x^2 (1+4\gamma))^{1/4} \quad [45]$$

This expression provides a qualitative picture of the interference effect for this particular case; the diffraction effect is proportional to the inverse square root of the distance behind the body, is exponential with the depth ratio d , proportional to the waterplane area and the $3/2$ power of the dimensionless wave number, and proportional to the inverse $1/4$ power of the wave encounter parameter.

The values of the poles for the coincident wave problem are shown in Figure 2, and these may be used to assess the relative values of the contributions of the poles to the total solution. For the previous example, the value of q_3/k is 1 for the entire range where the body heads into the waves; q_4/k is always greater except when encounter γ becomes large, where its contribution approaches that of the q_3 pole. For the overtaking range, the q_3 pole is also smaller and so it contributes the major portion of the result except when γ becomes large. An expression similar to Equation [45] could be obtained for this case by placing

$$q_3(0)/k = (1+2\gamma-(1+4\gamma)^{1/2})/2\delta^2 = ((1-\delta)/\delta)^2 \quad [46]$$

into

$$\tilde{\varphi} \approx \hat{\Delta}(q_3, -1 + \pi/4) \quad [47]$$

and comparing the result with Equation [4] with $\alpha = \pi$ and $\xi = 2d$.

The entire range of values of γ has now been covered, with the exception of the region where γ approaches $1/4$; at these points, the contributions from q_1 and q_2 become infinite, but in different ways at different places. When $\gamma > 1/4$ but approaching, the q_1 and q_2 contributions, previously eliminated, must be considered to obtain the way in which these contributions become singular. For $\gamma < 1/4$ but approaching, the q_1 and q_2 contributions become singular in quite another way. These two cases are considered next.

$$\epsilon = 4\gamma - 1$$

The q_1 and q_2 integrals require special consideration as the stationary phase point approaches the integration interval. With the small parameter ϵ defined by

$$\epsilon = 4\gamma - 1 \quad [48]$$

The integrals depending on q_1 and q_2 may be written

$$\begin{aligned} g_n &= \int_{\theta}^{\pi/2} d\theta \, w(\theta) e^{i\bar{\xi}q(\theta)} / (1 - 4\gamma \cos \theta)^{\frac{1}{2}} \\ &= \int_{\theta_1}^{\theta_2} + \int_{\theta_2}^{\pi/2} = \int_{\theta_1}^a + O(\bar{\xi}^{-1}) \end{aligned} \quad [49]$$

where θ_1 is defined by Equation [21]. For small ϵ ,

$$\theta_1 \approx (2\epsilon)^{\frac{1}{2}} \quad [50]$$

and Equation [49] may be reduced to

$$g_n = (w(\theta_1)/(\gamma \sin \theta_1)^{\frac{1}{2}}) \int_0^{(\theta_2 - \theta_1)^{\frac{1}{2}}} d\theta e^{i\bar{\xi}q(\theta_2 + \theta_1)} \quad [51]$$

where a change of variable has been employed. Evaluation of the integral results in

$$g_n = \pm i\delta^2 w(\theta_1) e^{i\bar{\xi}q_{1,2}(\theta_1)} / (4k\gamma^2 \bar{\xi} \sin \theta_1) \quad [52]$$

which gives the way in which the g functions are singular as $\epsilon \rightarrow 0$.

The results for the g_n functions are, then, for $\xi - \bar{\xi} < 0$

$$g_n = \tilde{U}(q_1, \theta_1, A_n, +1, +\pi/2) \quad [53]$$

where

$$\tilde{U}(q_m(\theta), \theta, A_n, u, v) = \hat{U} \chi \quad [54]$$

with

$$\chi = \cos(\bar{\eta}q_m(\theta) \sin \theta) \delta^2 / 4k\gamma^2 (\pi |\bar{\xi}| \epsilon / q_m(\theta) (1 - 4\gamma)^{\frac{1}{2}})^{\frac{1}{2}} \quad [55]$$

and for $\xi - \bar{\xi} > 0$

$$\begin{aligned} \xi_n = & \tilde{U}(q_2, \theta_1, A_n, +1, -\pi/2) + \\ & + \hat{U}(q_3, 0, B_n, -1, +3\pi/4) + \hat{U}(q_4, 0, B_n, -1, -\pi/4) \end{aligned} \quad [56]$$

The wave potential function is defined

$$\tilde{V}(q_m(\theta), u, v) = \hat{V}(q_m(0), u, v) \tilde{X} \quad [57]$$

which is accurate for small θ_1 . The wave potentials then become, for $\bar{\xi} < 0$,

$$\tilde{\Phi} = \tilde{V}(q_1(\theta_1), +1, \pi) \quad [58]$$

and for $\bar{\xi} > 0$,

$$\tilde{\Phi} = \tilde{V}(q_2(\theta_1), +1, 0) + \hat{V}(q_4(0), -1, -3\pi/4) + \hat{V}(q_3(0), -1, +\pi/4) \quad [59]$$

$$\underline{\epsilon = 1 - 4\gamma}$$

The q_1 and q_2 contributions are also singular in the four regions where $\gamma < 1/4$ but approaching $1/4$. These four particular points occur at

$$\delta = ((2 - \epsilon)^{\frac{1}{2}} - 1)/2 \quad [60]$$

for the body heading into the waves, and at

$$\delta = (1 \pm (\epsilon)^{\frac{1}{2}})/2$$

and

$$\delta = ((2 - \epsilon)^{\frac{1}{2}} + 1)/2 \quad [61]$$

for the body overtaking the waves.

The evaluation of Equation [25] was carried out numerically for small values of ϵ to determine the behavior of the g_n functions. The results are, for $\xi - \bar{\xi} < 0$,

$$g_n = \bar{U}(q_1, 0, A_n, +1, 0) \quad [62]$$

where

$$\bar{U}(q_m(\theta), \theta, A_n, u, v) = \hat{U} \bar{X} \quad [63]$$

with

$$\bar{X} = (4q_m(\theta)\epsilon^{\frac{1}{2}}|\bar{\xi}|/\pi)^{\frac{1}{2}} \sin(e^{13\pi/2}/q_m(\theta)|\bar{\xi}|(\pi\epsilon)^{\frac{1}{2}}) \quad [64]$$

and for $\xi - \bar{\xi} > 0$,

$$g_n = \bar{U}(q_2, 0, A_n, +1, 0) + \hat{U}(q_3, 0, B_n, -1, +3\pi/4) + \hat{U}(q_4, 0, B_n, -1, -\pi/4) \quad [65]$$

In a similar fashion, with the wave potential function defined as

$$\bar{V}(q_m(0), u, v) = \hat{V} \bar{X} \quad [66]$$

the wave potential becomes, for $\bar{\xi} < 0$,

$$\tilde{\varphi} = \bar{V}(q_1(0), +1, +\pi/2) \quad [67]$$

and for $\bar{\xi} > 0$,

$$\tilde{\varphi} = \bar{V}(q_2(0), +1, +\pi/2) + \hat{V}(q_4(0), -1, 3\pi/4) + \hat{V}(q_3(0), -1, +\pi/4) \quad [68]$$

The representation of the diffraction-altered wave field is now complete. The ordinary type of stationary phase representations hold throughout the range of γ except there $\gamma \approx 1/4$, at which points either the corrections of Equations [54] and [57], or of Equations [63] and [66], must be applied.

WAVE SPECTRUM RESPONSE

The results of the previous sections can be used, together with an appropriate wave-number spectrum, to define a mean wave alteration parameter, a_m , in the presence of a spectrum of waves.

An example of how this would be carried out, the incident wave height reduction factor of Equation [45] for the head-on-encounter case will be used. By defining the parameter

$$y = 2kd \quad [69]$$

Equation [45] may be rewritten

$$a = \bar{A} y^{3/2} e^{-2y/dL^2} (\pi x d (1 + 2 \text{Fr } y^{1/2})/2)^{1/2} \quad [70]$$

The ratio of the difference of wave amplitude p and reference wave amplitude p_{∞} to the reference amplitude p_{∞} is then

$$p - p_{\infty}/p_{\infty} = a \quad [71]$$

The ratio of energy densities is given by

$$E/E_{\infty} = (p/p_{\infty})^2 \quad [72]$$

The alteration in local spectral density, or mean wave height alteration parameter a_m , is then

$$\begin{aligned} a_m &= 2 \int_0^{\infty} dk (E - E_{\infty})/L \\ &= 2 \int_0^{\infty} dk E_{\infty} (a^2 + 2a)/L \end{aligned} \quad [73]$$

E_{∞} may be approximated by (see Reference 10)

$$\begin{aligned} E_{\infty} &= bL^3/16k^3 & k > k_0 \\ &= 0 & k < k_0 \end{aligned} \quad [74]$$

where the value of b is taken as .008. The final expression for a_m then becomes

$$\begin{aligned}
 a_m = & \bar{A}^2 b \int_{2k_0 d}^{\infty} dy e^{-4y} / (1 + 2 Fr y^{1/2}) / \pi x d L^2 \\
 & + \bar{A} b (2d/\pi x)^{1/2} \int_{2k_0 d}^{\infty} dy y^{-3/2} e^{-2y} / (1 + 2 Fr y^{1/2})^{1/2} \quad [75]
 \end{aligned}$$

Evaluation of this expression can be carried out numerically for specific values of x , d , \bar{A} , k_0 , and L .

The general problem which would, for example, consider the case of following waves, is rather more difficult; a multiple term expression for a , or, where γ is close to $1/4$, an expression with logarithmic components, is required. Since the final expression for a_m must itself be evaluated numerically, a general numerical procedure where the appropriate forms of the potential $\tilde{\phi}$ are used in the different regimes would seem to offer the best solution to this complex problem. Because of the behavior of the wave potentials near the $\gamma = 1/4$ singular points, and because of the logarithmic and square root nature of the singularities, integration over them to obtain the result for a wave spectrum can, in principle, be carried out; actual numerical evaluations are yet to be done.

CONCLUSIONS

The coincident wave-body problem chosen to illustrate the diffraction effect is seen to possess a multiplicity of problems precluding a simple qualitative result. The case with the body

heading into the waves, with $\gamma > 1/4$ does, however, allow an approximate result which illustrates the way in which a mean wave height alteration can be defined.

Equations [44] and [45] summarize these approximate results. Using Equation [25] for the head-on encounter case, Equation [45] can be rewritten

$$e^{-8dk}/(1+2\delta) = a^2 \pi x / 16 (\bar{A}/L^2)^2 k^3 \quad [76]$$

This relationship is shown graphically in Figure 3, where values of the modified alteration factor, the right side of Equation [76], are plotted against the quantity dk for values of the body velocity parameter δ from 0 to 2.5. As noted previously, as δ becomes large, the contribution from the q_4 pole becomes significant for this case, as shown in Figure 2. Care must also be taken, when applying these results, to recall the phase shift given by Equation [4'] when computing the wave diffraction effect, and that the alteration factor a in the same equation multiplies incident wave potential.

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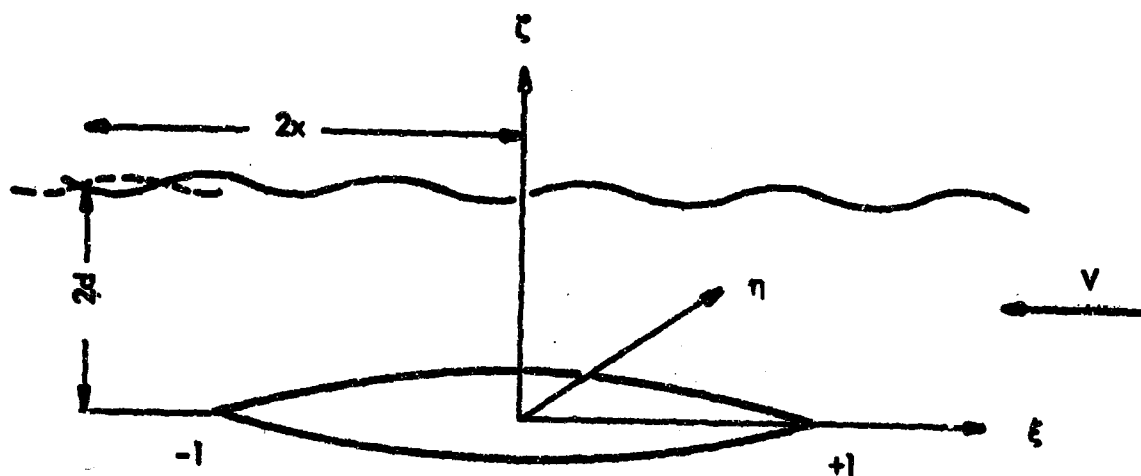
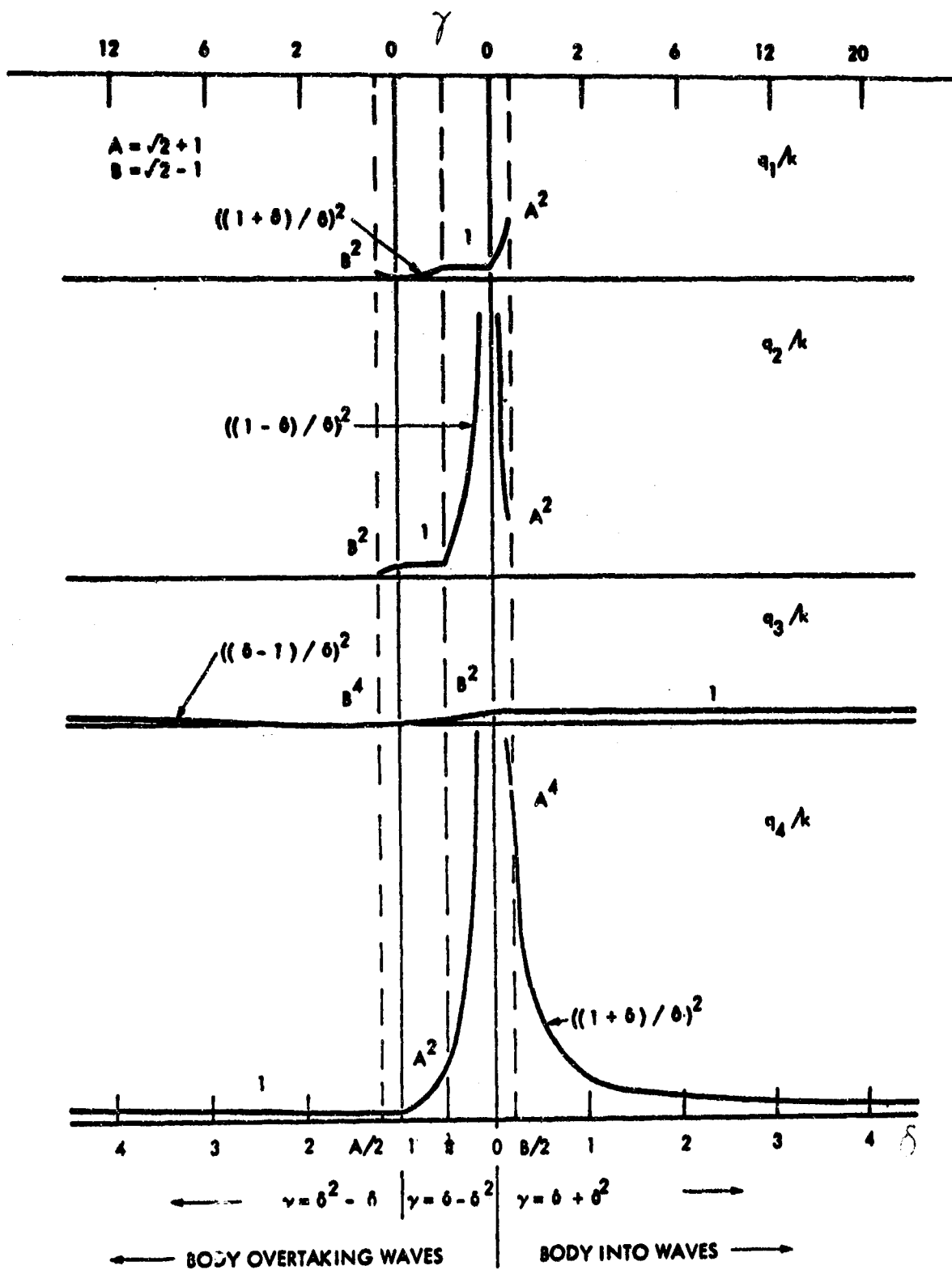


FIGURE 1 - COORDINATE SYSTEM (Unit length = $\frac{1}{8}$ body length)

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FIGURE 2 - POLE VALUES FOR COINCIDENT WAVE PROBLEM

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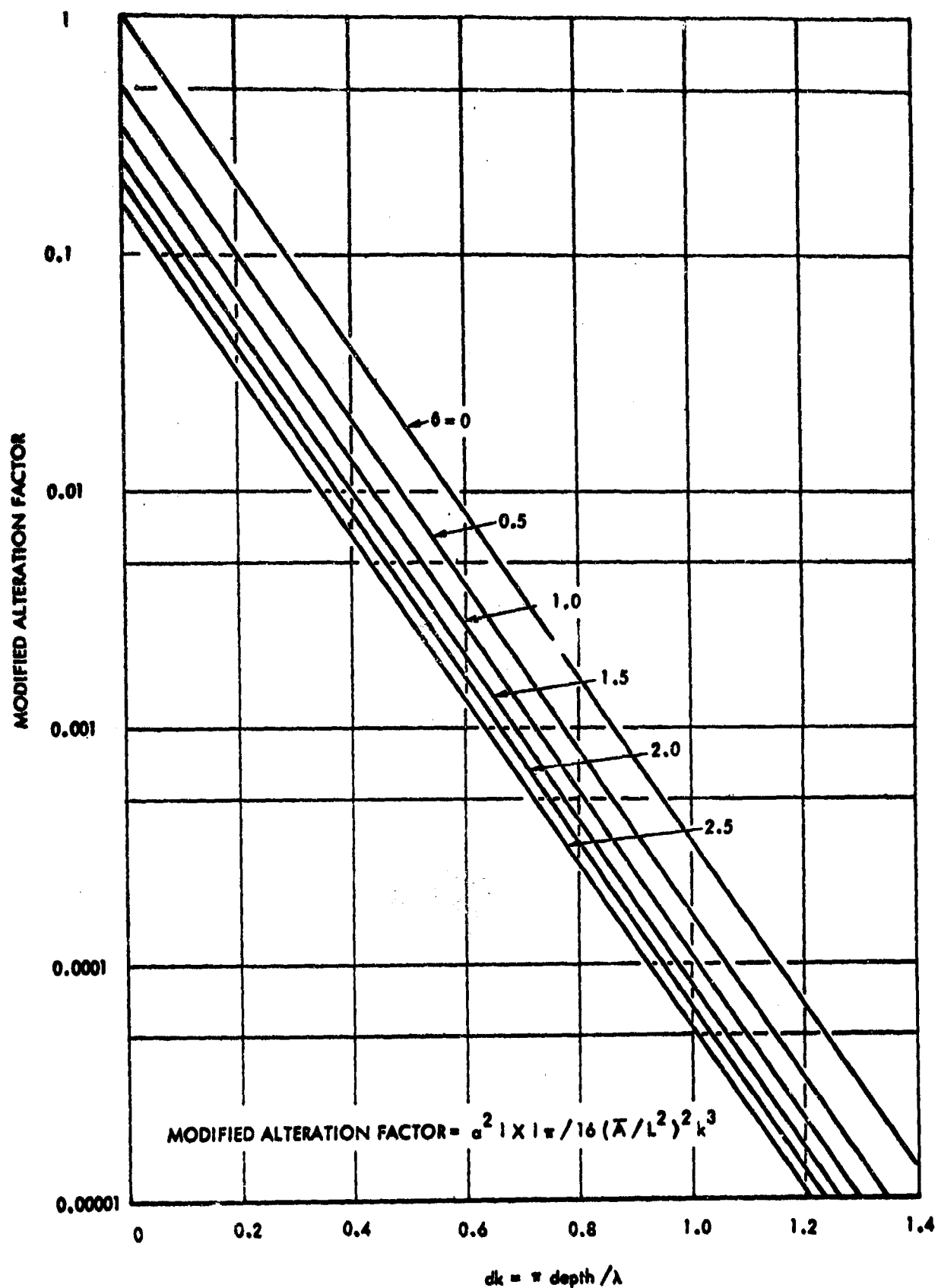


FIGURE 3 - MODIFIED ALTERATION FACTOR VS DEPTH AT VARIOUS SPEEDS

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